Orbispace uniformizations of sub-hyperbolic maps and their iterated monodromy groups

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Bonded orbit equivalence

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The orbit equivalence is *continuous* if there exists a continuous (i.e., locally constant) associated cocycle. It is *bounded* if the cocycle can be chosen to take a finite number of values (as a function of x) for every $g_1 \in G_1$.

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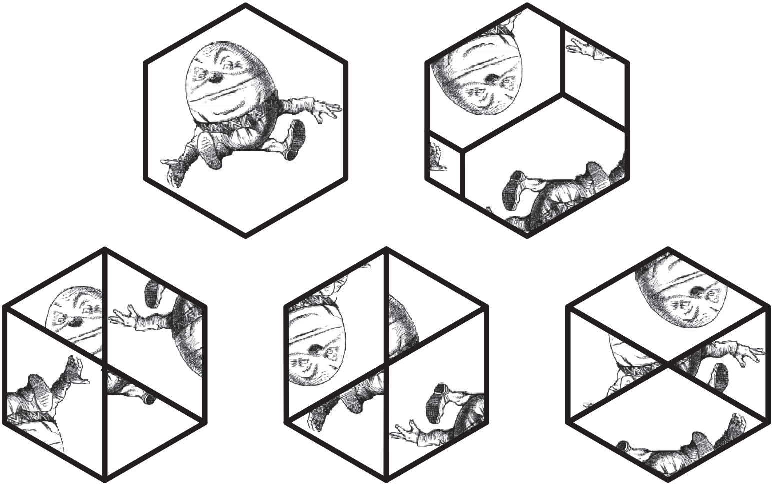
First examples of simple groups of subexponential growth were constructed using this method.



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We say that a correspondence $f, \iota : \mathcal{M}_1 \longrightarrow \mathcal{M}$ is *expanding* if $\mathcal{M}$ is compact and there exists a metric on $\mathcal{M}$ with respect to which f is a local isometry and ι is locally contracting.

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Here a faithful action of G on X^ω is *self-similar* if for every $g \in G$ and $x \in X$ there exist $h \in G$ and $y \in X$ such that $g(xw) = yh(w)$ for all $w \in X^\omega$.

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There are two natural orbispaces $\mathcal{M}, \mathcal{M}_1$ and a correspondence $f, \iota : \mathcal{M}_1 \longrightarrow \mathcal{M}$, where on the underlying topological spaces f coincides with the limit dynamical system, and ι with the identity map.

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If we remove all points of the post-critical set P_f, we get a correspondence $f, \iota : \widehat{\mathbb{C}} \setminus f^{-1}(P_f) \longrightarrow \widehat{\mathbb{C}} \setminus P_f$ of trivial orbispaces.

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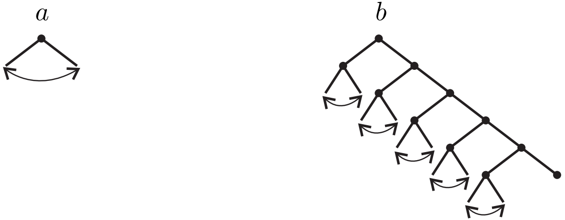
All self-similar contracting groups with this limit dynamical system (conjugate to the tent map) have been classified and constitute a class of groups defined earlier by Z. Šunić. All groups in this family are of intermediate growth, except for the dihedral group.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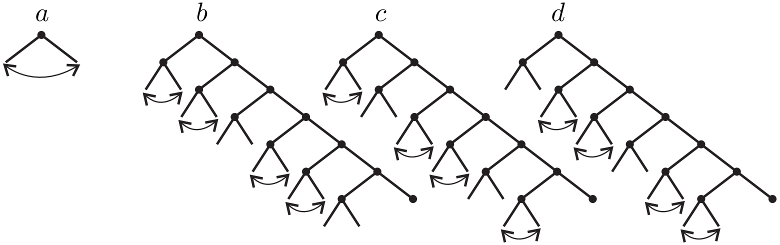
a
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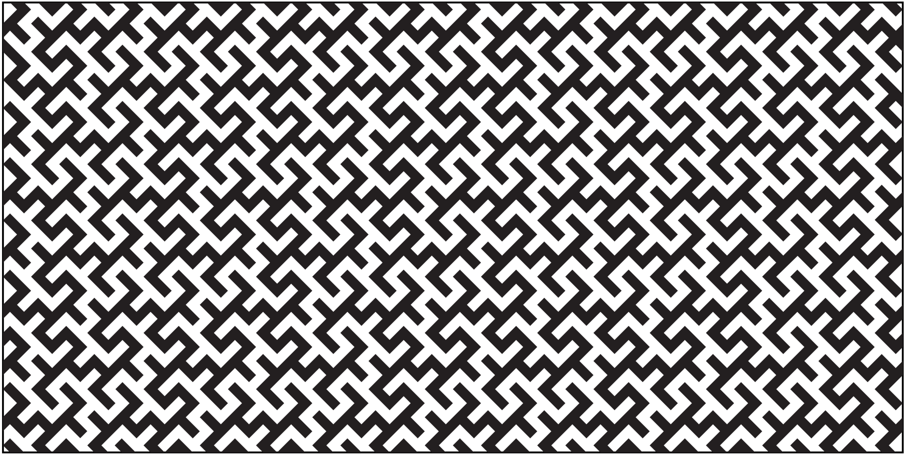

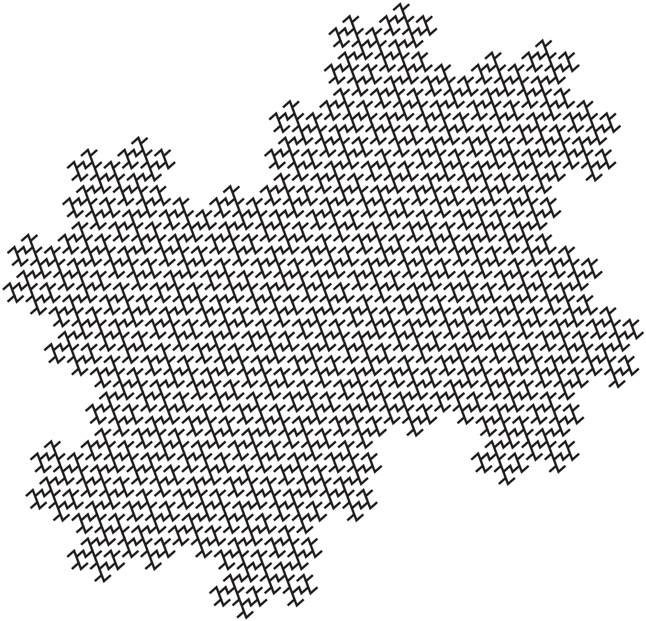

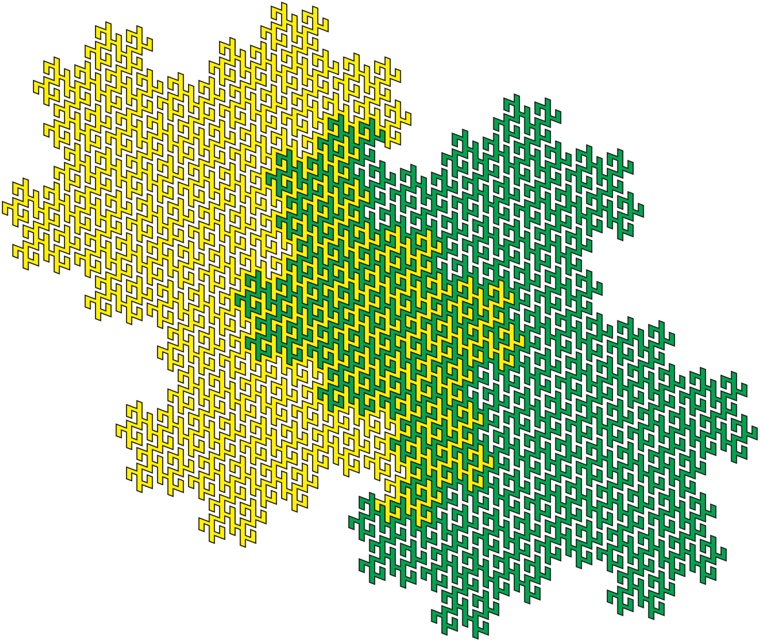

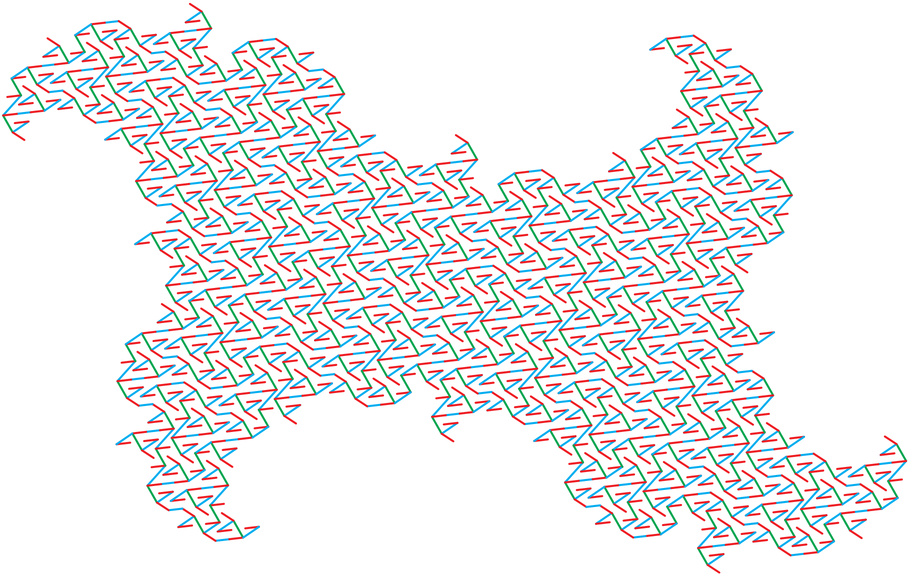

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